

So again the equinoctial year 1894-95 commences on March  $22^{\text{d}} + (1 - 0.972065)$ , or on March  $22^{\text{d}}.027935$  Greenwich Mean Time.

II. To find the mean equinoctial time corresponding to Greenwich mean noon on any given day:—

From the day of the year, increased if necessary by 365, subtract the number N, and affix the fraction for the year from column 4.

Thus: Required the equinoctial time at Greenwich mean noon on May 13, 1878.

Here the day of the year corresponding to May 13 would be 132, and in the equinoctial year 1878-79 N is 81, and the fraction of equinoctial time  $0^{\text{d}}.847521$ .

Therefore,  $132^{\text{d}} - 81^{\text{d}} + 0^{\text{d}}.847521$ ; or  $1878^{\text{y}} 51^{\text{d}}.847521$  is the mean equinoctial time at Greenwich mean noon on the day in question.

Again: What will be the equinoctial time at Greenwich mean noon on February 5, 1893?

The day of the year corresponding to February 5, 1893, is 35, and in the equinoctial year 1892-93, N between January 1 and the following vernal Equinox is 80, while the fraction of equinoctial time to be added is  $0^{\text{d}}.456497$ .

Hence,  $35^{\text{d}} + 365^{\text{d}} - 80^{\text{d}} + 0^{\text{d}}.456497$ , or  $320^{\text{d}}.456497$ , is the mean equinoctial time at Greenwich mean noon on February 5, 1893, which is required.

NOTE.—Should it ever be deemed expedient to reintroduce “mean equinoctial time” to a place in the *Nautical Almanac*, it would not be necessary to assign to it a column in a page for each month, as under former arrangements previous to 1876. All that would be requisite would be to give, in a note at the foot of the Table “Day of the Year, &c.,” the number N and the fraction of equinoctial time to be employed, both before and after the vernal Equinox of the current year, with a brief explanation of how they were to be applied; this last being given in the “Explanation of the Articles” at the end of the work.

A very slight alteration of the present arrangement of the type in the Table referred to, and a corresponding brief addition to the “Explanation,” would permit this.

Meadowbank, Melksham,  
1877, April.

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*Description of an Improved Diagram for the Graphical Solution of Spherical Triangles, applicable to the questions arising out of the Spheroidal Figure of the Earth, treated in the Paper read before the Society Nov. 10 ult., and further illustrated by the case of the Prediction of Occultations.* By F. C. Penrose, Esq.

The Table, which should have accompanied the paper referred to, giving the values of  $n$ —namely the distance between the centre of the Earth and the foot of the normal in different latitudes—and

for the correction of the Tabular Parallax when referred to the same point, is as follows. The Horizontal Equatoreal Parallax is taken as equal 57'. When more or less than this, the figures below should be adjusted proportionably :—

Latitude.	Correction to Polar Distance. Subtract from P.D.	Correction to the Equatoreal H.P. Add to H.P.
0	0	0
5	2.01	0.10
10	4.01	0.35
15	5.97	0.80
20	7.89	1.34
25	9.74	2.06
30	11.52	2.88
35	13.22	3.80
40	14.81	4.75
45	16.29	5.76
50	17.64	6.76
55	18.85	7.72
60	19.93	8.60
65	20.85	9.40
70	21.61	10.15
75	22.21	10.70
80	22.65	11.15
85	22.90	11.40
90	22.99	11.49

The diagram attached to the paper referred to, although it answers its purpose sufficiently well for working the spheroidal corrections, is not so well adapted for the general solution of a spherical triangle, because the lines representing the different zenith distances diverge so much from one another that it becomes troublesome to interpolate, and it offers but little facility for the converse problem of obtaining altitudes from given hour angles and polar distances; moreover, when the angle at the Moon is small, it does not suffice for determining the angle with sufficient accuracy.

The new diagram avoids these imperfections. The arrangement of the lines due to the different zenith distances is much more equable, so that interpolations can be effected with tolerable certainty.

This also applies to the interpolations required for using a diagram prepared for a particular latitude for latitudes within a few degrees of it.

If diagrams similar to the one submitted were made for every fifth degree of latitude, they would suffice, as I find by trial,

within limits of error of about  $30'$ , to determine any zenith distance from a given hour angle and polar distance, or for performing the converse operation of finding the hour angle or polar distance from the other two given data: and with more accuracy, of course, if a larger scale were adopted. I exclude, of course, unfavourable cases; such, for instance, as are produced by ill-conditioned triangles, which would even be avoided when logarithmic calculation is used.

This diagram belongs to latitude  $51^{\circ} 30'$ , and is, in fact, a projection of a sphere on a method not much unlike that of Mercator. It gives the relation between the three sides of a spherical triangle and one of the angles. Two of the sides and the angle may vary indefinitely, but one side is constant for the particular diagram. The angle referred to is adjacent to the constant side. For instance, if the constant side be supposed to be the co-latitude, the vertical measurements give polar distances and the oblique curved lines give zenith distances; but for general use I speak of the latter as the base, and the former as the side, and of the angle as the included angle—that is, included between the constant and the variable side. For instance, it will be seen by inspection of the diagram, that if we combine the vertical line drawn downwards from  $VI^h$  or  $90^{\circ}$  on the upper horizontal row of figures with  $50^{\circ}$  drawn across from the vertical column on the left, they will meet very near to the oblique line representing the base  $60^{\circ}$ . This, using the notation given above, is stated thus:—

The side  $50^{\circ}$  and included angle  $90^{\circ}$  have approximately for their base  $60^{\circ}$ . (This is true for  $59^{\circ} 48'$ .)

In a lunar triangle, we may thus obtain approximately the zenith distances of the Moon and star, and the parallactic angles of the two bodies. For this purpose generally two diagrams will be necessary, one adapted to the latitude and the other to the lunar distance. The following case, however, has been chosen as adapted to the one diagram herewith submitted.

In the figure below, if PSM, PZM be supposed to represent the measurements requisite for a lunar distance observation, the observer may be supposed to know

PZ	The co-latitude;
PM	The Moon's polar distance, by computation;
PS	The star's                    „                    „
SM	The lunar distance, by measurement;
ZPM	The Moon's hour angle, by computation;
ZPS	The star's                    „                    „

What are required are the two parallactic angles, viz. ZSM and ZMS, and the sides ZM and ZS, and also the angle PMS. The two sides indeed may, under favourable circumstances, be observed; but it is often impossible or inconvenient to do so. The diagrams enable all these quantities to be obtained by inspection with sufficient accuracy for general purposes, and,

should more exactness be required, the solution by this means offers a valuable check.

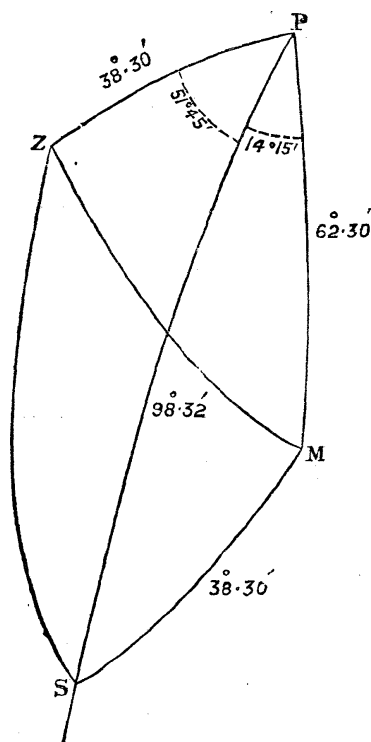
Thus all the data required for clearing the lunar distance may be determined.

Should the azimuth be required, it may be got, *mutatis mutandis*, as readily as the parallactic angle.

The simplest way of disposing of the spheroidal correction is by at once referring both the observation and the lunar triangle to the foot of the normal. When this is done, the geocentric lunar distance given in the three-hourly Arguments of the *Nautical Almanac* is, for any other latitude than the equator, too large or too small according as the angle PMS is acute or obtuse, and the excess or defect  $= n \cos D \cos M$ , where  $n$  is the quantity given above in the Table for the correction of the Moon's declination, and  $M$  is the angle PMS.

It seems, however, on several accounts more convenient to make this adjustment upon the measured distance, instead of the tabular argument, but in the contrary direction, so that when  $M$  is acute the distance must be increased, and decreased when it is obtuse, by the amount above stated.

As respects the increment to the parallax given in the Table, it should receive its due proportion when the parallax in altitude is computed.



As an example of reducing a lunar distance with the help of the diagram, let it be supposed that the altitudes have not been measured, but the distance only.

Then, in the lunar triangles PZM, ZMS, after computing the hour angles and polar distances of the Moon and star, we have—let us suppose—

PZ	=	38	30
PM	=	62	30
SM	=	38	30
PS	=	98	32
SPM	=	14	15
SPZ	=	51	45

In the spherical triangle PMS the use that is made of the given side SM is in the selection of the proper diagram, and the same applies to the use of the side PZ in the spherical triangle PZS.

Then, combining  $62^\circ 30'$  as a side with  $98^\circ 32'$  as a base, we obtain the included angle  $157^\circ$ . This is the angle PMS.

Then, in triangle PZS, using  $98^\circ 32'$  as a side and  $51^\circ 45'$  as an included angle, we obtain for the base PS  $74^\circ 30'$ .

Then, in triangle PZM, using  $62^{\circ} 30'$  as a side and  $ZPS + SPM = 66^{\circ}$  as an included angle, we get for the base  $54^{\circ}$ .

Then, in triangle ZSM, with  $74^{\circ} 30'$  as a side and  $54^{\circ}$  as a base, we obtain the star's parallax angle  $ZSM = 50^{\circ} 30'$ ; and lastly, in the same triangle, using  $54^{\circ}$  as a side and  $74^{\circ} 30'$  as a base, we obtain  $ZMS$ , the Moon's parallax angle  $= 112^{\circ}$ .

The corrections may now be made.

The effect of the increased value of parallax in altitude	}	"
$= 7'' \sin 54^{\circ} \cos 112^{\circ}$ , increasing the distance by .		

But the distance must be decreased, on account of the	}	15.3
term $n \cos D \cos PMS$ , by . . . . .		

Total correction . . . . .		13.2
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This example is not an imaginary case, but would have occurred in latitude  $51^{\circ} 30'$  S. and longitude  $4^h 34^m$  W. on Feb. 9th ult. in the distance of  $\alpha$  *Aquilæ* given in the *Nautical Almanac*. As the parallax then was  $54'$ , the amount above given must be reduced to  $12''.6$ . But this quantity would have affected the longitude by  $52''$ , and by more than 8 nautical miles in distance. And this is by no means an extreme case.

I showed in the former paper how this approximate determination of the altitudes and angles, &c., might serve to facilitate the accurate clearing of the lunar distance. I will proceed to show how it can be used to assist in predicting occultations and eclipses. At the bottom of the diagram is added a scale of logarithmic sines similar to that used on the slide rule, which will enable opposite sides or angles to be ascertained without recourse to another diagram.

As an example of its use, take the figure already given, and in the triangle PZM let it be required to find the angle PMZ. Since  $\log \sin PMZ - \log \sin PZ = \log \sin ZPM - \log \sin ZM$ , the difference on the logarithmic scale between PMZ and  $38^{\circ} 30'$  must equal the distance between ZPM and ZM. Measure then the interval between  $66^{\circ}$  and  $54^{\circ}$  on this scale and apply it to  $38^{\circ} 30'$ , and it will be found to reach to  $45^{\circ}$ , which is very nearly the true result. It should be observed that when the angles approach  $90^{\circ}$  this scale is not suited for accurate determination. At such points, however, the diagrams are at their maximum of efficiency.

For the purpose of predicting occultations, an additional diagram is prepared with certain requisite scales, especially one of minutes of arc for different values of parallax. It is suitable to all latitudes.

Having prepared the proper elements, lay down the Moon's course on the occultation diagram, and, with the hour angle and polar distance, find, on a diagram of zenith distances suited to the latitude, the zenith distance and parallax angle at the moment of conjunction: the former is required for ascertaining the amount of the Moon's parallax in altitude, and the latter for its direction with reference to the point C, which represents the



centre of the earth, or rather the foot of the normal belonging to the latitude.

When the position of the place for which the prediction is made is laid down for the moment of conjunction, it is seldom difficult to decide in what part of the Moon's path the phenomenon is to be looked for *within an hour*. Make, then, a new computation of the parallax in altitude and the parallactic angle, and lay down and join the points which have been determined. A scale is given for dividing any hourly segment of the Moon's path into 30 subdivisions, and there is little difficulty in dividing a corresponding hour interval on the line joining the two places of the terrestrial observatory [a segment always of an elongated ellipse, and which, it may be observed, is generally concave for north and convex for south declinations] into intervals representing 10<sup>m</sup> each. It is now easy to see which of the corresponding intervals are separated most nearly to the extent of the Moon's semidiameter, and what minute of time offers the best prediction. An example is given, namely, a prediction of the occultation of  $\rho$  *Leonis* Jan. 30, 1877, for Greenwich.

*Elements of Occultation.  $\rho$  Leonis, Jan. 30, 1877.*

Prediction for Greenwich.

	* 's R.A.			* 's Dec.	Geocentric
	h m s			approx.	diff.
G.M.T. of $\delta$	12	5	11	10 26 22	9 56
	0	1	59		27 36
	20	39	18		-0 18 to normal.
					27 18
G.S.T. of $\delta$	8	46	28		
	Hour angle 1 <sup>h</sup> 40 <sup>m</sup>				Parallax.
	Observer west of Moon.				geocentric 60 52
					+ 0 7
					60 59
					' "
					Moon's Motion in R.A. 34 10
					Eastwards 33 50
					And in Decl. southwards 16 8

With these measures of the Moon's motion lay down the Moon's course upon the occultation diagram, and HO will be the Moon's course during the hour preceding conjunction; and with the hour angle and declination above given obtain from the general latitude diagram, using them as an included angle and side, the zenith distance at conjunction.

This appears to be . . . . . 46 30  
The logarithmic sine scale will then }  
give the parallactic angle, about } 21 30

The parallax 60' 59" set off upon a radius drawn at an angle of 46° 30' from CZ will reach to  $\alpha$ .

The horizontal distance from  $a$  to  $ZC$  will be the parallax in altitude.

Set this off as  $CA$ , making an angle of  $21^{\circ} 30'$  with  $CZ$ .

$A$  is the observer's place at conjunction.

It is at once evident that the Moon's disappearance must have taken place more than an hour before conjunction. Take, then, an hour angle of  $3^h 10^m$  or  $1\frac{1}{2}$  hours before  $\odot$ , and with this as an included angle, and with the star's polar distance as a side, rework the observer's place. This will now be found to have

Zenith distance about  $56^{\circ} 30'$

and

Parallactic angle .  $33^{\circ} 35'$

With these measures determine  $Cb$  and  $CB$  in the same manner as  $Ca$  and  $CA$  were done before.  $B$  is then the observer's place  $1\frac{1}{2}$  hours before conjunction. Join  $AB$ —remembering that it is a portion of an elongated ellipse. Measure backwards upon the Moon's course  $Hh = \frac{1}{2} HO$ , and  $h$  becomes the Moon's place corresponding to  $B$ . By means of the diverging scales on the drawing, the line  $AB$  may be divided into 9 divisions each measuring 10 minutes, and  $Hh$  into 15 divisions each measuring 2 minutes. Having marked off as many of these divisions as may seem requisite, take the Moon's semidiameter suited to the parallax  $60'$ , and it will be found to reach across and bridge the interval between the two points on the lines  $Hh$  and  $AB$  due to the time  $1^h 14^m$  approximately before conjunction. By working in a similar manner the interval for the reappearance, it might be shown to agree most nearly with the time 8 minutes before conjunction;

These Greenwich times therefore are 10 51

and 11 57

These predictions are identical with those given in the *Nautical Almanac*, but are quite independent of it.

If the angle at the Moon's vertex for the observation is required, join the centre  $C$  with the observer's place at the time of the phase in question, and having drawn a circle representing the Moon, draw a straight line through its centre parallel to the former one. It will cut the Moon's circumference at the proper points for estimating the angle required. The figure gives the erect image, and must be allowed for accordingly when required for the inverted image.

### *On a Method of Destroying the Vibrations in a Mercurial Reflector.*

By *M. de Boë*, of Antwerp.

(Communicated by *Capt. Wm. Noble*.)

*M. de Boë* (Anvers) pense avoir perfectionné le système d'observation des fils de la lunette méridienne sur le bain de mercure.